Contribution

Leveraged ETFs: A Risky Double That Doesn't Multiply

by Two by William J. Trainor, Jr., Ph.D., CFA, and Edward A. Baryla, Jr., Ph.D.

Executive Summary

- This study investigates the long-run return properties of leveraged "ultra-bullish" exchange traded funds (ETFs), which seek to multiply the daily performance of market indexes both on the long and short side.
- Although leveraged ETFs can multiply index returns by a specific amount on a day-to-day basis, long-run returns cannot similarly be multiplied by the leverage ratio due to the constant leverage trap and the lognormal nature of continuously compounded returns. (Fund issuers make no representations regarding potential long-run return performance.)
- Over 50 leveraged ETFs have come to the market in just one year. An examination of these funds relating to major stock indexes shows that, on average, leveraged funds are meeting their specified daily leverage targets. But there is significant volatility in meeting this target on any given day.
- Using Monte Carlo simulation, it is found that a typical 2x leveraged stock ETF is likely to only magnify the index return by 1.4 times on an annual basis for holding periods out to ten years. But the risk as measured by the standard deviation of returns remains doubled, while the magnitude of extreme negative returns can quadruple.
- The paper also compares leveraged ETFs with buying an index fund using a simple margin account. It finds the leveraged ETFs a superior long-term alternative due to their relative lower cost.
- Given the risk/return trade-off leveraged ETFs offer, long-term investors should be considerably wary. Extreme swings in value are inherent in this type of fund.
- Leveraged ETFs seem to be a useful tool for short-term investors or traders who are willing to extend themselves by making a market call.

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As investors' appetite for risk has increased in the pursuit of increasing returns, leveraged exchange traded funds (ETFs) have found their niche. More than 50 have been launched in just one year, collecting $6 billion in assets (Lydon 2007). Their popularity stems from the fact that these funds seek to multiply the daily performance of market indexes both on the long and short side. They are able to do this by employing a variety of leveraging techniques including the use of futures and options among other instruments. The most common multiplier tends to be two times the index, although the multiplier ranges across funds from 1.25 to 2.5. So-called "ultra-bullish" funds are those designed to multiply the market's normal return, while an "ultra-short" bearish fund is designed to deliver a multiple of the opposite of the market's return. This paper focuses on ultra-bullish funds.

Fund companies that issue leveraged ETFs point out that their investment goal is only to lever the returns by a specific amount on a day-to-day basis, not over an extended period. Despite this caveat, investors may be under the misguided belief that these leveraged funds will also multiply long-term returns by the same day-to-day amount, or at least come approximately close. But this is not the case due to two effects.

The first effect, which is fully disclosed in the fund prospectus, is the constant leverage trap. In brief, this "trap" refers to a magnified compounding problem. The second effect, which has likely not been considered, is the fact that accumulated wealth from continuously compounded daily returns is lognormally and not normally distributed. This effect becomes more pronounced over an extended period and further compromises the long-run leverage multiplier of these funds.

In fact, using Monte Carlo simulation, this study shows that even over a one-year horizon, the expected median annual return for 2x leveraged ETFs is only 1.4 times the index, while the risk as measured by the standard deviation of returns does indeed double. In their defense, we do find that, on average, leveraged ETFs are able to magnify daily gains and losses at least close to their touted leverage ratios. But even on a day-to-day basis, there is significant volatility associated with these leverage ratios.

As an alternative to leveraged ETFs, we compare the expected long-run performance of buying an index fund using a simple margin account. We find that the long-run expected median returns are very similar to the leveraged ETF. Since the margin strategy does not employ constant leverage, and yet has similar returns, we find that the lognormality of compounded returns is the overriding factor for why leveraged ETFs cannot maintain their leverage ratios over extended periods of time. We suggest that given the risk/return trade-off of leveraged ETFs, most investors should be wary of this investment vehicle. Extreme swings in value are inherent in this type of fund.

The Constant Leverage Trap

The constant leverage trap has been well documented by both fund companies and the popular press (Gullapalli and Salisbury 2007, Yates and Kok 2007). Constant leverage requires an investor to maintain an exact percentage of leverage over the entire time horizon. If one is using a margin account, this requires an investor to buy in a rising market and sell in

a declining market. The "trap" occurs because this type of strategy magnifies the compounding problem. The compounding problem is based on the mathematical principle that the geometric mean of a series of numbers is lower, the greater the variance of the numbers. Using leverage magnifies the variance of the returns. A simple example of these effects will demonstrate why, in general, a 2x leveraged ETF can double the daily movements of an index but cannot simultaneously double its longer run returns.

Assume you invest $100 in both a market index and a 2x leveraged ETF. The index falls 10 percent, and thus the leveraged ETF falls 20 percent. The next day the index increases by 20 percent and the leveraged ETF increases by 40 percent. Your portfolio value in the index falls to $90, and then increases to $108, for a gain of 8 percent. Note the average return is 5 percent each day, yet your portfolio only averages 4 percent each day. This is the compounding problem. In the leveraged ETF, your portfolio value falls to $80, but only increases back to $112, for a gain of 12 percent. Instead of averaging 10 percent each day, your portfolio only averages 6 percent each day. The leverage ratio for each day is two, but over the two days is only 1.5 (12 percent divided by 8 percent). Thus, the compounding problem is magnified by the use of constant leverage.

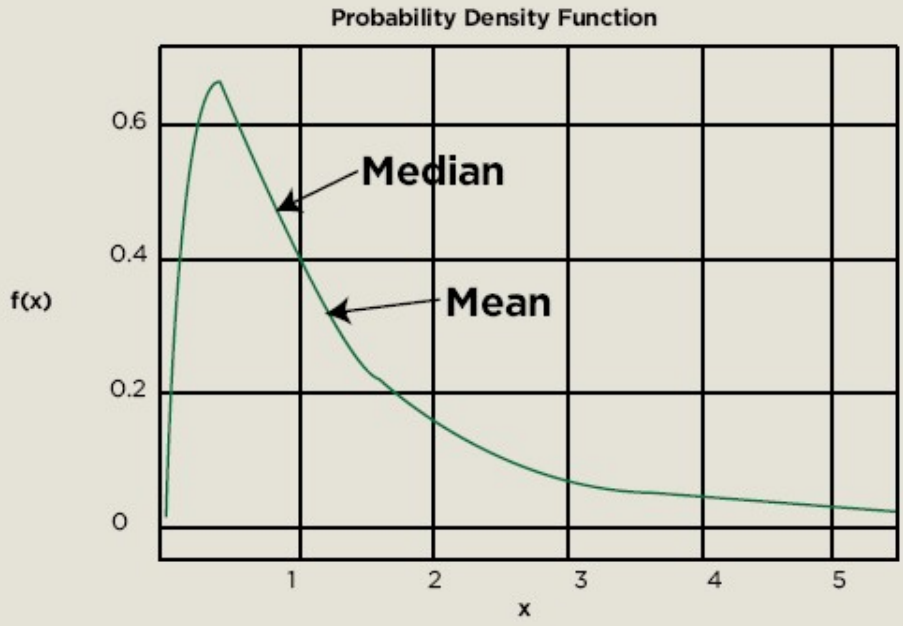
Why Lognormality Matters for Leveraged ETFs

Although it is a statistical fact that compounding random returns causes long-term returns to be lognormally rather than normally distributed, the effect of this is not always understood. As an example, assume there is an even chance of a 20 percent or –10 percent return. The average return per period is then 5 percent. After two periods there are three possible outcomes: 44 percent (20 percent twice); 8 percent (20 percent, –10 percent or –10 percent, 20 percent); and –19 percent (–10 percent, –10 percent). The average cumulative return for the two periods is 10.25 percent. But note that only one outcome of four exceeds this average: the 44 percent scenario. The compounding causes the distribution of the discrete returns to become positively skewed. In addition, although there are fewer returns greater than the mean, they exceed it by a greater amount, on balance, than the more plentiful below-average returns (Kritzman 2003). After several periods of compounding, the distribution of returns conforms to a lognormal distribution.

To graphically demonstrate this issue, Figure 1 shows a typical lognormal distribution. As one may note, returns are positively skewed, which causes the median return to be less than the mean return. This is particularly pronounced with leveraged ETFs due to their much greater standard deviation of returns. The greater the standard deviation, the more skewed the distribution. The effect of this over time not only causes expected mean long-run returns to be less than double, but median returns—that is, those at which there is at least a 50 percent chance of reaching—are even less. Hughson, Stutzer, and Yung (2006) also point this out and make a convincing argument for why cumulative median returns are more relevant. (For a mathematical discussion, see Appendix A.)

Figure 1: Standard Lognormal Distribution

A doubling of the mean and standard deviation causes the distribution to become more skewed, which in turn causes the mean to diverge even further from the median.



Leveraged ETF Historical Results

Before simulating what future long-run results can be expected, it is of interest to examine how leveraged ETFs have performed so far. On June 18, 2007, the CBS.Marketwatch.com Web site showed the performance returns for the

Proshares Ultra S&P Fund (symbol SSO) and the returns on the S&P 500 index. The Ultra Fund is designed to double the daily performance of the S&P 500 index (see Table 1).

Table 1: Performance Ratio of ProShares Ultra S&P Fund Versus S&P 500 Index

	Ultra S&P 500 Fund	S&P 500 Index	Performance Ratio
1 Day	1.29%	0.65%	1.98
1 Week	3.27%	1.69%	1.93
4 Weeks	0.98%	0.76%	1.29

The performance ratio is the return on the Ultra Fund divided by the return on the S&P 500 index. This ratio shows that the fund is able to meet the 2x daily performance goal. The actual ratio falls just short at 1.98, but given the volatility of daily returns, one might say that a one-day performance ratio of 1.98 is well within reason.[1] One can also see that the performance ratio deteriorates over time. In fact, the four-week performance ratio fell to 1.29 over this particular period. Figure 2 shows this dramatic drop is primarily the result of a large decline in the S&P 500 during the first week of June. The greater the volatility, the lower the performance ratio will be for any given expected return.

Figure 2: Proshares Ultra 2x Versus the S&P 500

From a practical perspective, one might consider that the casual observer would note that an investor in this leveraged fund did achieve returns that are superior to the S&P 500 index, and thus tout its validity as a bullish investment vehicle. This might indeed be a viable argument for those who adopt a short-term trading strategy. But it may not be the case for those who would like to consider leveraged ETFs as a long-term investment vehicle.

Table 2 shows the one-year market price return for ProShares leveraged ETFs as of October 31, 2007, as reported at www.proshares.com. These funds are designed to double the daily performance of the Nasdaq, the Dow Jones Industrial Average, S&P 500, and the S&P Mid-Cap 400, respectively. The table compares fund performance with the matching index funds offered by Fidelity (Powershares, Diamonds, and SPDRs) and Barclays (ishares).

Table 2: ProShares Returns Versus Matching Index Fund

ProShares Fund	Return	Matching Index Fund	Return	Performance Ratio
Ultra QQQ (QLD)	53.66%	Powershares QQQ (QQQQ)	29.65%	1.81
Ultra Dow 30 (DDM)	28.48%	Diamonds (DIA)	17.52%	1.63
Ultra S&P 500 (SSO)	20.72%	SPDR (SPY)	14.33%	1.45
Ultra Mid-Cap 400 (MVV)	25.33%	I-Shares 400 Index (IJH)	16.43%	1.54

The performance ratio again shows that return performance cannot be maintained over time. In this case, the best one-year performance ratio is the Ultra QQQ at 1.81, while the Ultra Mid-Cap only manages 1.54. It should be re-emphasized here that the fund issuer makes no representation regarding the fund's long-term ability to exceed the market by a given percentage. The fund focus is on daily returns. But this subtlety may be lost on the casual investor. Indeed, such an investor who examines this table would probably consider an investment in the Ultra QQQ as a sure-fire path to riches, as the year-over-year gain is over 50 percent. In a consistently rising market, this may well be the case. Unfortunately, markets do correct and bear markets emerge from hibernation, at which time this type of investor would learn the meaning of risk too late (for examples, see below).

Leveraged ETFs are a relatively new investment product. But fund families have also issued regular mutual funds that attempt to magnify market performance. These funds do offer some evidence on possible long-term performance. Table 3 shows returns for Profund's Classic and Ultra OTC Fund.[2] The Classic OTC Fund is designed to match the Nasdaq 100, while the Ultra OTC Fund is designed to double the Nasdaq 100 on a daily basis.

Table 3: Ultra OTC Fund Versus Classic OTC Fund

	Ultra OTC Fund	Classic OTC Fund	Performance Ratio
Year to Date	17.83%	10.16%	1.75
1-Year	37.41%	20.91%	1.79
3-Year	12.04%	8.61%	1.40
5-Year	9.64%	8.74%	1.10

Again, the casual observer might note that the investor did achieve returns superior to its bogey, making this fund a good potential long-term investment. Of course, that opinion might differ if the market suffered an extended downturn. It is interesting to note that the Ultra-to-Classic performance ratio moves closer to one as the time horizon becomes longer.

The same basic results held for Profund's Ultra Small Cap, Ultra Mid-Cap 400, and their Ultra Bull fund, which are targeted to double the daily returns of their Russell 2000, S&P 400, and S&P 500 index funds, respectively. In each case, performance ratios fell over time, starting from 1.6 to 1.8 for one-year results, and falling to the 1.5–1.6 range by five years.

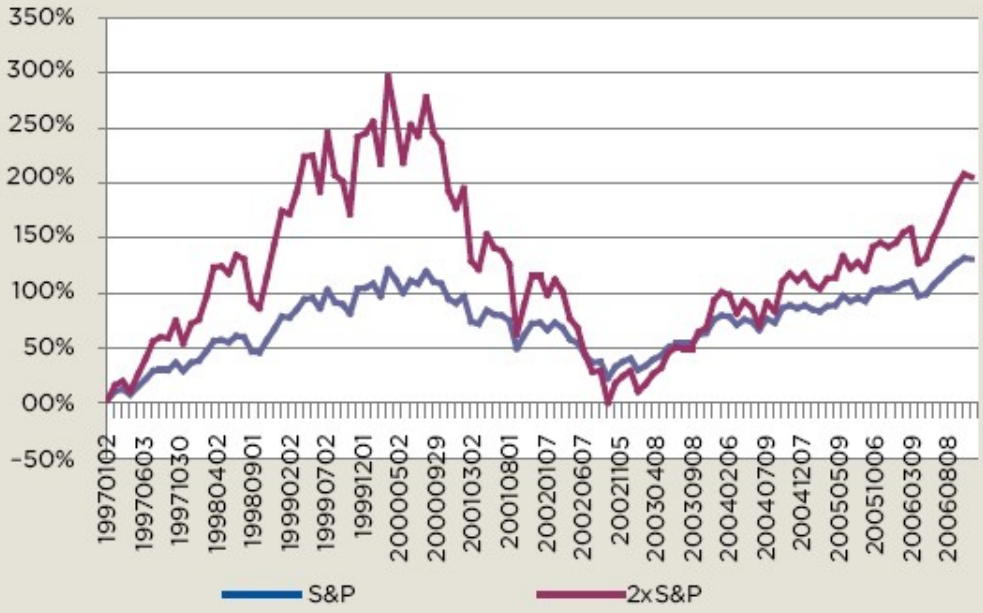
Given the historical evidence, one is hard pressed to deny that leveraged funds, whether ETFs or mutual, can offer a significant boost to portfolio returns. But this represents one price path. For comparative purposes, one also can consider the Rydex Investments (www.rydexfunds.com) Dynamic OTC H Class Fund, which seeks to double the daily performance of the Nasdaq 100. Someone who invested in this fund on its inception date of May 24, 2000, has suffered an average annualized total return of –26.90 percent as of October 31, 2007.

But based on the five years ending October 31, 2007, the average annual total return for this fund is 27.77 percent. Thus, even though the fund has done quite well since the 2001 market crash, those who invested before the market crash are still down a staggering –26.90 percent on an annual basis! Put another way, that means they still have less than 15 percent of what they had over six years ago. Clearly, being invested in leveraged funds during a sharp market decline such as 2001 can more than offset years of above-average returns.

To reinforce this point, Figure 3 shows what a hypothetical 2x leveraged fund would have done over the ten-year period from January 1997 to December 2006. With the bull market in full force, a 2x leveraged fund would actually have had a cumulative return of almost 2.5 times a standard index fund by mid-2000. Yet less than two years later, all of the leveraged fund's gains are completely wiped out. This would be particularly painful for investors who entered the market in the early part of 2000, as demonstrated by the actual results of the Rydex Dynamic OTC fund as described above. Yet by the end of 2006, the hypothetical leveraged fund would have ended up with a cumulative return that outperformed the standard index fund by 1.59 times. Clearly, a leveraged fund can provide spectacular results, both on the positive and negative side.

Figure 3: Cumulative Return for a Hypothetical 2x Leveraged S&P 500 Fund

Assumes a start date of January 1, 1997, and continuing to December 2006 based on historical daily returns.



What Long-Run Results Can Be Expected

The historical evidence presented does provide a measuring benchmark. But examining just one historical return path is not advisable for estimating expected returns. Additionally, with the short history of leveraged ETFs, this procedure is even more prone to error. Fortunately, Monte Carlo simulation allows us to create thousands of return paths to estimate not only what is likely to occur in the future, but also to see the range of returns that may occur.

In this study, we create 30,000 ten-year daily return sequences to analyze what could occur for non-leveraged and 2x leveraged funds.[3] We calculate one-, three-, five-, and ten-year means, medians, and 95 percent confidence intervals. In addition to the leveraged fund results, we also analyze an account using 50 percent margin. We assume the margin account is rebalanced once a year. The margin rate is set at the historical average 30-day T-bill rate of 4.2 percent. For the leveraged fund, cost of implementation is assumed to be 2 percent, which is the average expense ratio of leveraged ETFs. Costs for both the margin and leveraged accounts are charged daily. We also compute performance ratios for each of the metrics presented in Table 4. This allows us to make a standardized comparison of the risk and return characteristics of the presented alternatives.

Table 4: Monte Carlo Simulation Comparing No Leverage, 2x Leverage, and Margin Account

Annualized Return		Mean	Median	Standard Deviation	Lower CL	Upper CL
1 Year	No Leverage	10.7%	9.1%	19.9%	-23.0%	34.9%
	2x ETF	20.1%	12.9%	44.3%	-43.7%	73.7%
	Performance Ratio	1.88	1.42	2.22	1.90	2.11
	50% Margin	17.2%	13.9%	39.4%	-49.6%	65.0%
	Performance Ratio	1.61	1.53	1.98	2.16	1.86
3 Years	No Leverage	9.5%	8.8%	11.4%	-11.1%	37.0%
	2x ETF	15.1%	12.5%	24.1%	-25.1%	78.5%
	Performance Ratio	1.59	1.42	2.11	2.26	2.12
	50% Margin	13.0%	13.2%	28.3%	-38.9%	71.8%
	Performance Ratio	1.37	1.50	2.48	3.50	1.94
5 Years	No Leverage	9.4%	8.9%	8.8%	-6.7%	16.5%
	2x ETF	14.2%	12.6%	18.4%	-17.3%	29.1%
	Performance Ratio	1.51	1.42	2.09	2.58	1.76
	50% Margin	11.3%	11.6%	22.8%	-27.2%	37.2%
	Performance Ratio	1.20	1.30	2.59	4.06	2.25
10 Years	No Leverage	9.1%	8.9%	6.2%	-2.4%	14.4%
	2x ETF	13.5%	12.6%	12.8%	-9.7%	33.4%
	Performance Ratio	1.48	1.42	2.06	4.04	2.32
	50% Margin	11.2%	10.9%	13.2%	-14.01%	30.4%
	Performance Ratio	1.23	1.22	2.13	5.84	2.11

Using Monte Carlo simulation and creating 30,000 ten-year daily return paths, annual returns are computed for 1-, 3-, 5-, and 10-year holding periods based on a daily return of 0.04 percent and a standard deviation 1.1 percent. Results using no leverage, 2x leverage, and a margin account are presented. The study shows the mean, median, standard deviation, 95 percent confidence interval, and the performance ratio as measured by the 2x ETF or margin portfolio results divided by the no-leverage results. The margin rate is assumed to be 4.2 percent and the leverage fund is assumed to have an expense ratio of 2 percent. The 2x levered ETF median return performance ratio averages 1.4 times the index return.

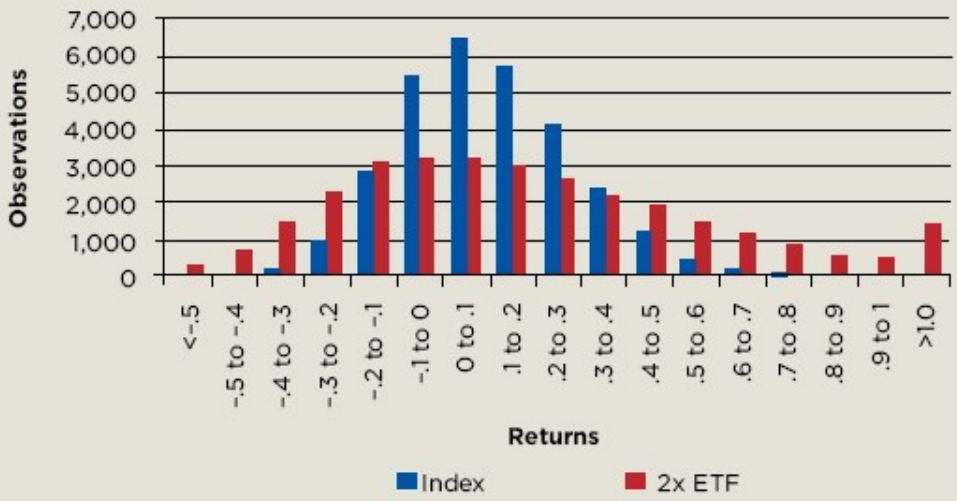
Although leveraged ETFs are directed at multiple indexes and sectors, the main ones are the S&P 500, the Dow Jones Industrial Average, the Nasdaq 100, and the Russell 2000. Historically, the DJIA has reliable daily return data going back to 1906, Center for Research in Security Price (CRSP) has S&P daily data starting in 1962, and Nasdaq data starting in 1972. The average daily returns range from 0.033 percent to 0.047 percent, while the standard deviations range from 0.93 percent to 1.2 percent across these indexes. Based on the average values from these indexes, we simulate return paths using a daily return of 0.04 percent and a daily standard deviation of 1.1 percent. This corresponds to an approximately 10 percent annualized market return and a 20 percent annualized standard deviation.[4]

Table 4 shows the results based on our Monte Carlo simulations for one-, three-, five-, and ten-year investment horizons. For a one-year horizon, the standard index fund has an expected median return of 9.1 percent, the 2x ETF has a median return of 12.9 percent, and the margin account has a median return of 13.9 percent. The performance ratio shows that the 2x ETF falls well short of doubling the index median return as it is only 1.4 times the return of the index.

Some Evidence to Support Investing in ETFs

There is some evidence to support investing in a leveraged ETF. The mean return for a one-year horizon is 20.1 percent compared with the index mean return of 10.7 percent. Unfortunately, there is only a 40 percent chance of attaining this mean due to the lognormality issue. This probability becomes even less with longer run horizons. This is illustrated in the histogram in Figure 4, which shows 30,000 annual return observations. As one can see, the 2x ETF has a flatter distribution that is positively skewed. While the index return has 17,532 one-year return paths between –10 percent and 20 percent, the 2x ETF has only 9,436. On the other hand, the 2x ETF has substantially more negative possibilities: 7,751 return paths less than –10 percent compared with only 3,937 for the index. But it is the more positively skewed return of the 2x ETF that causes its mean to be so much greater than its median return. For the 2x ETF, there are 1,385 return paths that result in a return greater than 100 percent. The index portfolio has only six. These extreme returns positively bias the mean. Although there is a small chance of doing very well, the more likely outcome for an investor is the median return.

Figure 4: Histogram of 2x ETF Versus Index



The number of annual return observations for the index and a 2x leveraged ETF from 30,000 Monte Carlo simulations are based on a one-year time horizon. Ten percent incremental return ranges are created ranging from less than –50 percent to greater than 100 percent. The 2x ETF return distribution is flatter and more positively skewed, resulting in a median return of only 12.9 percent compared with its mean return of 20.1 percent. In contrast, the index median return is 9.1 percent, while its mean return is only slightly higher at 10.7 percent.

Table 4 also shows that for a one-year horizon, the margin account actually has a slightly higher median return even though the cost of this strategy is assumed to be more than twice that of the ETF. But it should be noted that the margin account is only rebalanced once a year, and on average, the leverage actually decreases throughout the year as the fund grows. Regardless, because the margin strategy does not employ constant leverage, and yet has similar returns, it seems evident that the lognormality of compounded returns is the overriding factor as to why leveraged ETFs are unable to maintain their performance ratios over extended periods of time.

For longer-term horizons, the performance ratio of the ETF median returns relative to the index returns remains approximately the same. In fact, by three years, the performance ratio converges to approximately 1.4 and the expected leveraged ETF median annualized return stabilizes at 12.6 percent. Thus, for long-term investors, expected returns are indeed higher. But there is significantly more risk relative to return because the standard deviation for leveraged ETFs is consistently more than twice as much as the standard deviation for the underlying index. Furthermore, by examining the confidence interval, one can see that the lower limit's annualized return for a leveraged ETF is approximately two times lower than the index for a one-year horizon. By five years, the lower limit of the ETF is more than 2.5 times lower on an annualized basis, and by ten years, it is more than 4 times lower.

On the other hand, as seen earlier in Figure 3, it is possible for leveraged ETFs to actually outperform the index over time by more than their leverage ratio. This possibility also can be seen in Table 4 by looking at the one-year upper confidence interval. The performance ratio of 2.11 shows that, on average, when the market does extremely well, the 2x

ETF ends up with more than twice the return of a standard index fund. Although there is less than a 2.5 percent chance of this occurring, in an extreme bull market where the market is up a vast majority of days, a leveraged ETF can pay off handsomely.

We also present performance ratios for the standard deviation of returns. For every holding period, the standard deviation is more than double for the leveraged ETF scenario when compared with the no-leverage scenario. Also, in all but the one-year holding period, the same holds for the margin scenario. As neither the mean nor median performance ratios are this high, it would seem that as a stand-alone asset, the risk of holding either the leveraged fund or the margin portfolio exceeds the benefit when compared with the no-leverage scenario. It is also worth noting that the standard deviation performance ratio is less for the leveraged scenario when compared with the margin scenario, except for the one-year holding period. Thus, if one had to choose whether to invest in the leveraged fund or to create a similar fund using a margin account, this evidence suggests that the leveraged fund provides greater long-term return for less risk.

Conclusion

The evidence presented in this paper shows that investors can secure higher returns using leveraged ETFs. But even for annual returns, investors should not expect their returns to be twice that of the index. Due to the lognormality of continuously compounded returns, along with the constant leverage trap, the most likely scenario for the typical 2x leveraged fund is to only attain 1.4 times the index return. But this reduced leverage gain is accompanied by twice the risk as measured by the standard deviation of returns, and has the possibility of extreme negative returns. Thus, to secure a leveraged ETF's higher return, investors must take on an excess amount of risk for the expected level of return. Although leveraged ETFs have become quite popular, risk-averse investors should remain wary.

One tactic that might prove beneficial, however, is to use leveraged ETFs as a trading vehicle. Granted, this is a purely speculative play, but market participants who enter and exit positions on a daily or even multi-monthly basis could easily increase their returns by making a proper market call, and then using the leveraged ETF as the investment vehicle of choice. In addition, for those investors who use margin to invest in standard index funds, leveraged ETF funds appear to be a superior long-term alternative due to their lower cost relative to borrowing on margin.

Endnotes

1. Twelve additional funds from various firms were examined on a weekly basis through June of 2007, starting with each fund's inception date, which ranged from June 2004 to June 2006. On average, these funds are achieving their leverage ratios in the very short run, which they are designed to do.
2. Profund's Classic and Ultra OTC fund have recently been renamed the Nasdaq 100 and Ultra Nasdaq 100 respectively.
3. We actually create three runs of 10,000 ten-year daily return paths to make sure values converged. Average values generally varied less than 0.2 percentage points across runs. Based on this, we simply combined the three 10,000 runs to make one large data set consisting of 30,000 ten-year daily return paths.
4. Results are based on the actual annual returns from the simulations. Daily returns are not annualized.

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Appendix A

This appendix explains how to convert daily returns into expected mean values and confidence intervals for long time horizons. In addition, it shows why doubling the daily mean and standard deviation causes long-run returns to be less than double. First, one must convert daily returns and standard deviations into their continuous counterparts. The equations to do this are the following:

$$\mu_c = \ln(1 + \mu_d) - \sigma^2_c/2 \quad (1)$$

$$\sigma_c = \sqrt{\ln[\sigma^2_d/(1+\mu_d)^2 + 1]} \quad (2)$$

where

μ_c = continuous expected return
μ_d = discrete expected return
σ_c = continuous standard deviation
σ^2_d = discrete variance.

Note that the continuous mean is a function of the variance. This is because the lognormal distribution is not symmetric and the value of the variance affects the spread of the distribution, which in turn affects the mean. Once these values are computed, the standard deviation is further adjusted based on the time horizon. If dealing with annual data, the standard adjustment is to divide the continuous standard deviation by the square root of the number of years that one is forecasting. Confidence intervals can then be derived using standard normal distribution tables, and continuous values are converted back into their discrete counterparts (Trainor 2005).

As an example, assume that the daily index expected return is 0.04 percent with a 1.1 percent standard deviation. These numbers are based on the daily returns from the Center for Research in Security Price's (CRSP) value-weighted index from 1926 to 2006. A 2x leveraged exchange traded fund will have exactly twice the same numbers. But after converting these numbers into their continuous counterparts using equations (1) and (2), the index fund will have a continuous return of 0.034 percent and a continuous standard deviation of 1.1 percent, while the leveraged ETF will have a continuous return of 0.055 percent and a continuous standard deviation of 2.2 percent. Because the magnitude of the variance affects the mean, the leveraged ETF has only 1.6 times the index's continuous return rather than twice the amount, yet will have twice the standard deviation. As one forecasts accumulated wealth into the future, this effect is compounded. This paper demonstrates the repercussions of this fact.